

MECHANICAL PROPERTIES OF FLUIDS

Introduction:

The branch of physics which deals with the study of fluids at rest is called **hydrostatics** and that branch of physics which deals with the study of fluids in motion is called **hydrodynamics**.



Hydrostatics



Hydrodynamics

Solid is a state of matter which has a definite shape and definite volume. **The liquid** another state of matter which has a definite volume but no shape of its own. It takes the shape of the vessel in which it is placed.



Solid



Liquid



Gas

The gas is also a state of matter which has no definite volume and shape. It takes the shape and volume of the entire vessel containing it. This behavior is accounted for the fact that the strength of forces between molecules of the solids is maximum, it is less in liquids and least in gases. Since the liquids and gases can flow, hence they are called **fluids**. Thus, fluid is the name given to a substance which begins to flow when external force is applied on it.

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Pressure of a liquid:

The normal force (or thrust) exerted by a liquid at rest per unit area of the surface in contact with it is called pressure of liquid or hydrostatic pressure.

Let F be the normal force acting on a surface area A of the object in contact with liquid. The pressure exerted by liquid on this surface of object is $P = \frac{F}{A}$

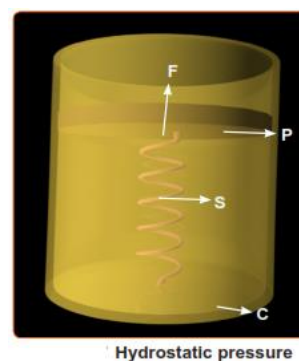
The smaller the area A on which the given force F acts, the greater is the impact of force i.e. pressure and vice-versa.

Unit of pressure: The SI unit of pressure is N/m^2 or Pascal and the CGS unit is dyne/cm^2 .

The dimensional formula for pressure is $\text{ML}^{-1}\text{T}^{-2}$.

Pressure is a scalar quantity, because at one level inside the liquid, the pressure due to liquid is exerted equally in all directions. It shows that a definite direction is not associated with the pressure due to liquid.

Measurement of hydrostatic pressure: A simple device used to measure the hydrostatic pressure is shown in figure. Here C is an evacuated cylindrical chamber of a metal, having an airtight frictionless piston P , which can slide inside the chamber. S is a metallic spring of known spring constant. Its one end is connected to piston and other end to the inner bottom face of chamber. This device is placed at a location inside the liquid, where pressure due to liquid is to be measured. The force due to liquid presses the piston downwards. The spring gets compressed. A restoring force comes into play in spring which will try to take the piston back.



In equilibrium position, the inward force (F) on the piston due to liquid becomes equal to the outward spring force ($= kx$), where ' r ' is the compression of the spring and ' k ' be the spring constant of spring. If A is the area of cross-section of the piston, then average pressure due to liquid at a location is $P_{av} = \frac{F}{A} = \frac{kx}{A}$ can be measured.

Some applications of pressure:

1. The bags and suitcases are provided with broad handles so that small pressure is exerted on the hand while carrying them.
2. Railway tracks are laid on large sized wooden, iron or cement sleepers, so that the thrust due to weight of train is spread over a large area. This reduces the pressure on ground which would prevent the yielding of ground.



Pressure on hand



Pressure on ground



Pressure on the surface

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3. Pins and nails are made to have pointed ends in order to have least area of contact between the pin and the given surface. Due to this, the pin if pressed, will exert high pressure on the surface, and hence will easily penetrate the surface.

DENSITY:

The density of a substance is defined as the mass per unit volume of the substance, i.e,

$$\text{density, } \rho(\text{Rho}) = \frac{\text{mass}}{\text{volume}} = \frac{m}{v}$$

The S.I. unit of density is Kg/m^3 and CGS unit is gm/cm^3 .

The dimensions of density are ML^{-3} .

Density is a positive scalar quantity.

A solid or liquid is generally incompressible, since density remains constant at all pressures. The gases are generally compressible, their densities vary with pressure.

The density of water at 4°C ($= 277$ Kelvin) is $1.0 \times 10^3 \text{ Kg}/\text{m}^3$

Relative density:

Relative density of a substance is defined as the ratio of its density of a substance to the density of water at 4°C .

$$\text{Relative density} = \frac{\text{density of a substance}}{\text{density of water at } 4^\circ\text{C}}$$

Relative density has no units and no dimensions. It is a positive scalar quantity.

Variation of pressure with depth:

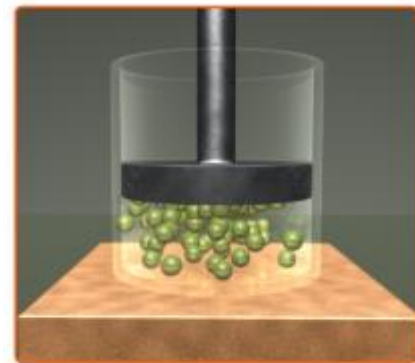
Consider a liquid of density ' ρ ' (Rho) contained in a vessel at rest. Let C and D the points inside the liquid at a vertical distance.

Imagine a cylinder of liquid with axis CD cross sectional area A and length h, such that the points C and D lie on the flat faces of the cylinder.

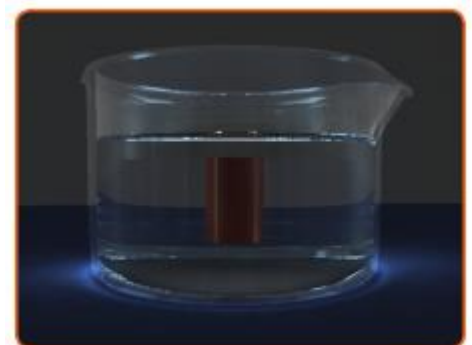
The mass of the liquid in the imaginary cylinder will be

$$M = \text{volume} \times \text{density} = Ah \times \rho = Ah \rho \rightarrow (1)$$

Let P_1 , and P_2 , be the pressures of liquid at points C and D respectively.



Gas Compressible



Variation of Pressure with Depth

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The liquid cylinder is under the action of following vertical forces.

(1) Force $F_1 = P_1 A$, acting vertically downwards on the top face of the cylinder,

(ii) Force $F_2 = P_2 A$, acting vertically upwards on the lower face of the cylinder,

(iii) Weight, $Mg = Ah \rho g$ of the liquid cylinder acting vertically downwards.

As the liquid is in equilibrium, the net force on it must be zero.

$$F_1 + Mg - F_2 = 0$$

$$\text{Or } P_1 A + Ah \rho g - P_2 A = 0 \quad \text{or } P_2 - P_1 = h \rho g \dots (2)$$

If points C and D lie at the same level in liquid,

then 'h' = 0, From (2),

$$P_2 - P_1 = 0 \text{ or } P_1 = P_2$$

This shows that the pressure is same at all points inside the liquid lying at the same horizontal plane.

From (2), we note that pressure difference between two points of liquid depends on

(i) The vertical distance between points C and D,

(ii) Density of liquid ' ρ ' and

(iii) Acceleration due to gravity ' g '.

If the point C is shifted to the top of the liquid surface, which is exposed to the atmosphere, then P_1 can be replaced by atmospheric pressure P_a .

If $P_2 = P$, then total pressure at point D is given by

$$P - P_a = h \rho g \text{ or } P = P_a + h \rho g \dots (3)$$

This shows that the total pressure P at depth ' h ' below the surface of liquid at rest, which is open to atmosphere is greater than atmospheric pressure by an amount $= h \rho g$. This excess of pressure at depth ' h ' in liquid $= P - P_a = h \rho g$. It is called gauge pressure at point D.

Thus, gauge pressure at a point in a liquid is the difference of total pressure at that point and atmospheric pressure.

Pressure exerted by liquid column:

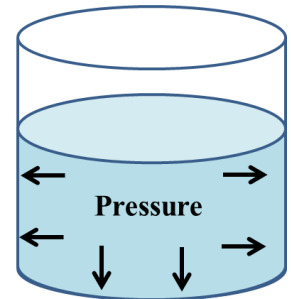
If $P_L =$ pressure due to liquid at point D, then $P_2 - P_1 = h \rho g \dots (4)$

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This shows that the pressure exerted by a liquid column of height 'h' is independent of area of cross section 'A' but depends upon the height 'h' of the liquid column and density ρ of the liquid.

Pascal's Law:

- Pressure of a fluid is not exerted only on a solid like the walls of the container or a piece of solid immersed in it.
- Pressure exists at every point within the fluid.
- The pressure applied to any part of the enclosed fluid at rest is transmitted undiminished to every portion of the fluid and to the walls of the vessel.
- Barometer is used to measure atmospheric pressure while manometer is used to measure pressure difference or gauge pressure.



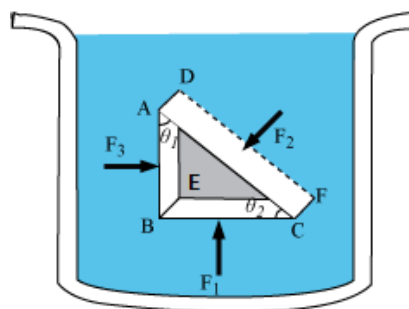
Pascal's law:

The pressure in fluid at rest is the same at all points if they are at the same height.

Verification of Pascal's law:

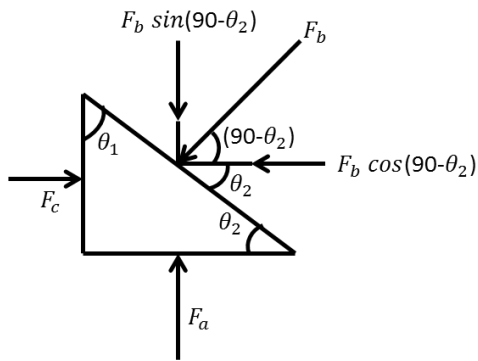
Consider a small element in the interior of a fluid at rest. The element ABC-DEF is in the form of a right-angled prism. Every part is considered at the same depth from the liquid surface and therefore, the effect of the gravity is the same at all these point.

Suppose the fluid pressure P_a, P_b and P_c on the faces BEFC, ADFC and ADEB respectively. The corresponding normal forces are F_a, F_b and F_c as shown in figure. Let A_a, A_b , and A_c be the respective areas of the three faces in various directions.



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By Newton's law, the force should balance in various directions.



Along horizontal

$$F_b \cos(90 - \theta_2) = F_c \text{ (By equilibrium)}$$

$$F_b \sin \theta_2 = F_c$$

Along vertical

$$F_b \sin(90 - \theta_2) = F_a$$

$$F_b \cos \theta_2 = F_a$$

$$A_b \sin \theta_2 = A_c \text{ and } A_b \cos \theta_2 = A_a \text{ (By geometry)}$$

$$\text{Thus } \frac{F_b \sin \theta_2}{A_b \sin \theta_2} = \frac{F_c}{A_c} ; \frac{F_b \cos \theta_2}{A_b \cos \theta_2} = \frac{F_a}{A_a}$$

$$\therefore \frac{F_b}{A_b} = \frac{F_c}{A_c} = \frac{F_a}{A_a}$$

Hence pressure exerted is same in all directions in a fluid at rest.

Thus, Pascal's law is verified.

Application of Pascal's Law:

Whenever external pressure is applied on any part of a fluid contained in a vessel, it is transmitted undiminished and equally in all directions. This is the Pascal's law of transmission of fluid pressure and has many applications in daily life.

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Basing on Pascal's law, devices such as hydraulic lift and hydraulic brakes are designed. In these devices fluids are used for transmitting pressure.

Hydraulic Lift:

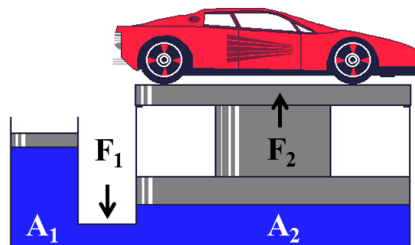
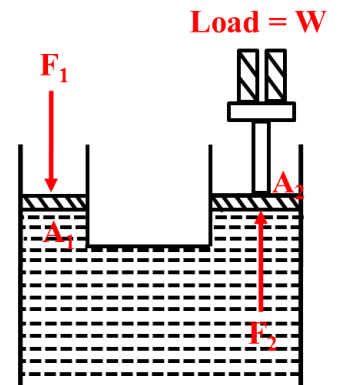
If F_1 is the force applied to the piston in the narrow limb, then there is a change in pressure.

$P = \frac{F_1}{A_1}$, This is transmitted undiminished to all points in liquid.

Change in pressure at A_2 is also P .

$$P = \frac{F_2}{A_2} \quad \text{Thus, } \frac{F_1}{A_1} = \frac{F_2}{A_2}, \quad F_2 = F_1 \times \frac{A_2}{A_1}, \quad F_2(W) = F_1 \times \frac{A_2}{A_1}$$

Thus, the force exerted on the second piston is increased by the factor equal to the ratio $\frac{A_2}{A_1}$.



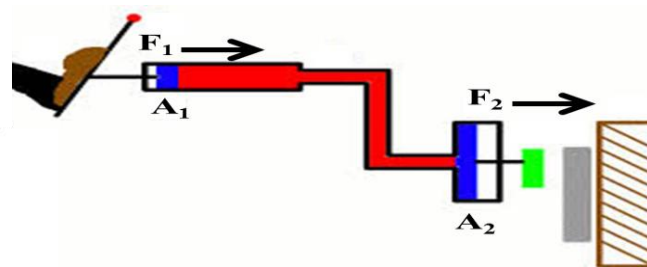
$$P_1 = \frac{F_1}{A_1} \quad P_2 = \frac{F_2}{A_2}$$

$$P_1 = P_2$$

According to Pascal's Law

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}, \quad \frac{F}{A} = \text{Constant}, F \propto A. \text{ If } A_1 < A_2, \quad \text{Then } F_1 < F_2$$

Hydraulic break:



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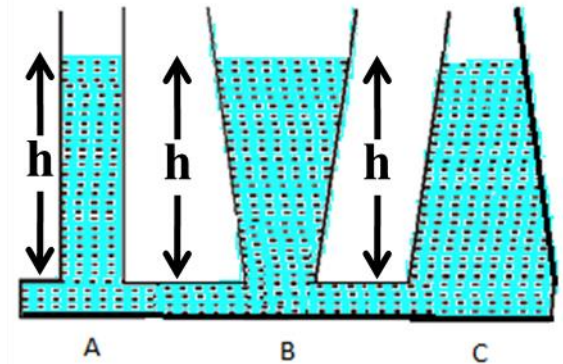
$$P = \frac{F}{A} = \text{constant}, F \propto A, \quad \text{If, } A_1 < A_2 \text{ then, } F_1 < F_2$$

If we apply less force over a small area, we can produce a large force over large area.

Hydrostatic paradox:

The shape of the vessel containing the liquid does not affect the pressure. This is known as Hydrostatic paradox.

It is found that all the vessels of different shapes, holding different amount of liquid upto the same height, the same pressure is recorded at the base of each of the vessel.

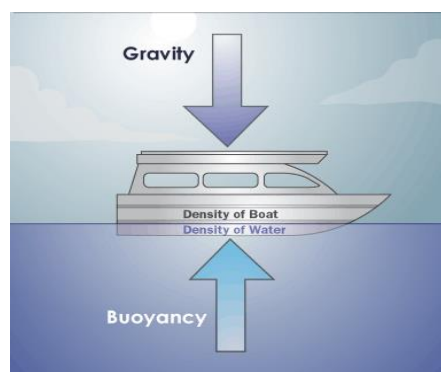
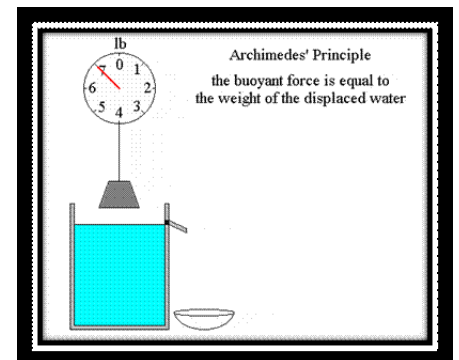


The pressure exerted by column of liquid on the base depends upon the height of the liquid and not on the shape of the containing vessels.

Archimede's principle:

Archimedes' principle states that the magnitude of the buoyant force is always equal to the weight of the fluid displaced by the object.

- When a body is immersed partially or wholly into a liquid, it experiences an upward thrust, which is equal to the weight of the liquid displaced by the body.
- Here, the upward thrust is called buoyant force. It acts through the centre of buoyancy which is centre of gravity of the displaced liquid.



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What is buoyancy?

Buoyancy is the ability or tendency of something to float in water or any other fluid.

Apparent weight = True weight – upthrust

- If a body of volume V and density d_1 is fully immersed in a liquid of density d_2 , then the true weight $W = Vd_1g$

Weight of displaced liquid = Vd_2g

Apparent weight $W' = Vd_1g - Vd_2g$

$$= Vd_1g \left(1 - \frac{d_2}{d_1}\right); W' = W \left(1 - \frac{d_2}{d_1}\right)$$

a) If $d_2 < d_1$, $W' > 0$ and the body will sink to the bottom.

b) If $d_2 = d_1$, $W' = 0$ and the body will just float or remain hanging at whatever position it is left inside the liquid.

c) If $d_2 > d_1$, $W' < 0$ i.e., upthrust will be greater than the true weight. The body will move partly out of the free surface of the liquid until the upthrust becomes equal to true weight W .

Laws of flotation:

a) The weight of the floating body is equal to the weight of liquid displaced.

b) Centre of gravity of the floating body and centre of gravity of the displaced liquid are in the same vertical line.

- If a body of volume V_1 and density d_1 floats in a liquid of density d_2 and V_2 is the volume of the body immersed in the liquid, then

$$V_1d_1g = V_2d_2g \quad , \quad V_1d_1 = V_2d_2$$

If the liquid is water, relative density or specific gravity = $\frac{v_2}{v_1}$

$$\text{Specific gravity of a liquid} = \frac{\text{loss of weight in liquid}}{\text{loss of weight in water}}$$

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- If W is the weight of a body in air, W_1 is its weight in a liquid and W_2 is weight of that body in water (in both cases completely immersed)

$$\text{relative density of liquid} = \frac{W - W_1}{W - W_2}$$

Streamline flow:

Fluid flow is categorized as,

- i) Stream line or laminar flow and ii) Turbulent flow.

i) Stream line or laminar flow:

If velocity of each particle at a point in a fluid does not change with time in magnitude and direction then the flow of the fluid is called stream line flow or steady flow or laminar flow.

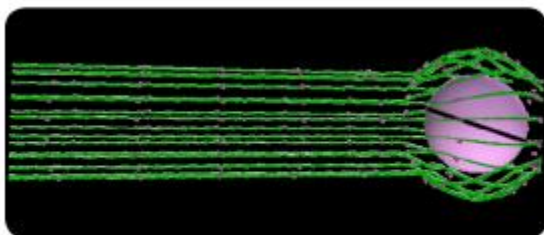
The line along which a particle of the liquid moves and the direction of which at any point give the direction of flow of liquid at that point is called stream line.

A stream line is a curve or a straight line such that at any instant the tangent to it at a point gives the direction of flow of the liquid at that point.

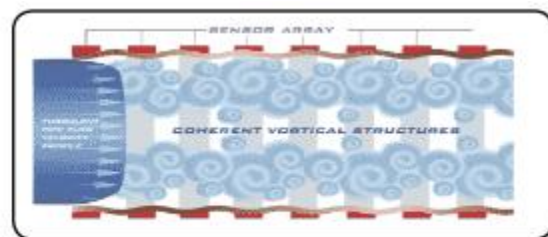
ii) Turbulent flow:

Turbulent flow is featured by eddy currents or eddies. Eddies absorb large amount of energy. In turbulent flow internal friction is much greater than that in streamline flow.

We consider four important characteristics of fluid in streamline or turbulent flow. These are as follows.



Stream line and laminar flow



Turbulent flow

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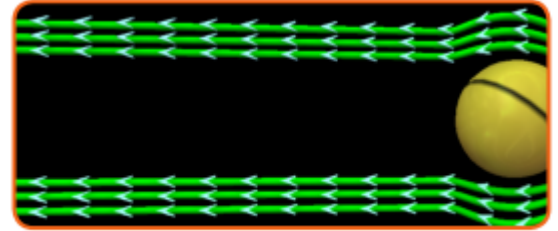
1. Fluid flow may be steady or non-steady (unsteady):

The velocity of the flow at each point in space remains constant with in time in a steady flow.

This does not mean that velocity should be the same at all points in space.

If velocity of the fluid at a point changes with time, the flow is called unsteady flow.

Beyond certain velocity, known as critical velocity, a steady flow becomes non-steady.

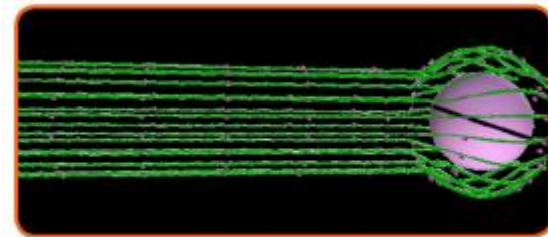


Fluid flow Steady

2. Fluid flow may be rotational or irrotational:

Flow is irrotational if there is no net angular momentum of fluid at each point. If a small paddle wheel is placed at any point in the fluid it would not rotate.

In rotational flow, at each point a net angular momentum is realized, and a paddle wheel would rotate if placed at any point in the flow.



Fluid flow rotational

3. Fluid flow may be compressible or incompressible:

If no change in density of the fluid is observed along the line of flow, then the flow is known as incompressible. Otherwise the flow is compressible.

In fact no material is truly incompressible; the flow of many fluids is such that variations in density are so small that they can be ignored.



Fluid flow compressible

4. Fluid flow may be viscous or non viscous:

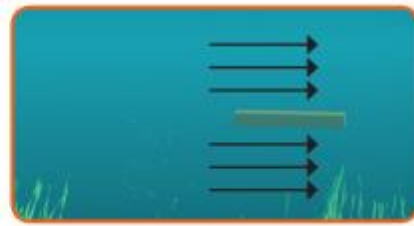
As a fluid flows, there exists an internal friction between the adjacent layers of the fluid and hinders the motion. This internal friction is known as viscosity. Viscosity results in

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dissipation of mechanical energy. The flow of a fluid is viscous when the effect of viscosity (internal friction) on the flow is considerable and cannot be ignored.



Fluid flow viscous



Fluid flow Non viscous

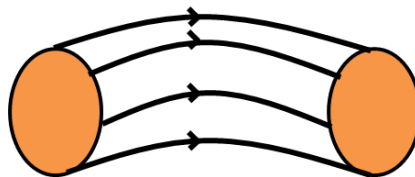
When viscous forces can be ignored, the flow can be considered as non-viscous.

Here in our study of fluid dynamics we assume that the flow is steady, irrotational, incompressible and non-viscous.

Tube of flow:

- A bundle of streamlines having the same velocity of fluid over any cross-section perpendicular to the direction flow is called tube of flow.

tube of flow



CRITICAL VELOCITY:

The critical velocity is that velocity of liquid flow, upto which its flow is streamlined and above which its flow becomes turbulent.

REYNOLDS NUMBER:

The critical velocity of a liquid is (a) Direct proportional to its coefficient of viscosity η ,

(b) Inversely proportional to its density d

(c) Inversely proportional to the radius ' r ' of the tube through which the liquid flows.

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If V_c is critical velocity, $V_c \propto \frac{\eta}{dr} \Rightarrow V_c = R \frac{\eta}{dr}$

Here the proportionality constant 'R' is known as Reynold's number.

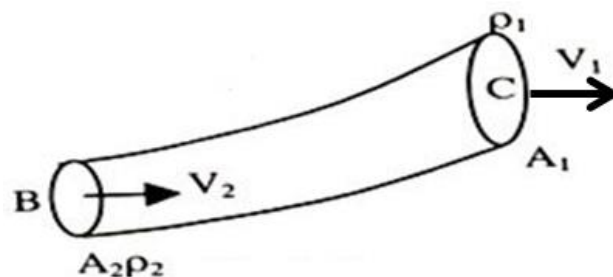
$$R = \frac{V_c dr}{\eta}$$

For narrow tubes R is about 1000. It is a pure numeric and independent of the system of units used. Fluids with the same value of R will be flowing similarly and have similar streamlines. The shape of the streamlines and direction of motion of a body in a viscous medium depends on the Reynold's number.

It is found that flow is streamline or laminar for R_e less than 1000. The flow is turbulent for $R_e > 2000$. The flow becomes 'unsteady' for R_e between 1000 and 2000. The critical value of R_e (known as critical Reynolds number), at which turbulence sets, is found to be the same for the geometrically similar flows.

Equation of continuity:

According to the principle of continuity, mass of fluid flowing per second entering into the tube is equal to mass of fluid flowing per second leaving out of the tube.



$$\begin{aligned} \text{Mass of fluid flowing per second} &= \frac{\text{Mass}}{\text{Time}} \\ &= \frac{\rho \times \text{vol}}{\text{time (t)}} \quad (\because \text{mass} = \rho \times \text{Volume}) \end{aligned}$$

$$\text{Volume} = \text{Area} \times \text{displacement}$$

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$$=A \times x \quad = \frac{A \times \rho}{t} \quad = A \times v \times \rho$$

Consider a liquid in a streamline flow through a tube of non-uniform area. The liquid enters with velocity V_1 at B and leaves with velocity V_2 at C. Let ρ_1 and ρ_2 be the densities of the liquid at B and C respectively.

$$\text{Mass of fluid flowing per second} = \frac{\text{mass}}{\text{time}}$$

Mass of incompressible fluid flowing per second through point B and C will be same

$$\rho A_1 v_1 = \rho A_2 v_2$$

$$A_1 v_1 = A_2 v_2 = \text{Constant}$$

$$A \times v = \text{constant}$$

This equation is called the equation of continuity.

$$\therefore V \propto \frac{1}{A}$$

If cross-sectional area is small, then the velocity of fluid increases & vice versa.

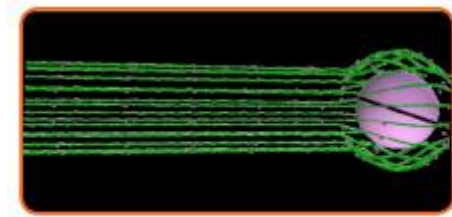
Limitations:

- Flow should be steady (streamline).
- Fluid should be incompressible & non-viscous.
- According to the principle of continuity mass is conserved.

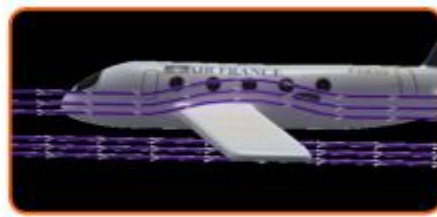
Bernoulli's Principle:

A small plastic ball can be made to float above a blowing jet of a fluid. Airplane wings and airfoils are designed in a particular shape. Smoke goes up the chimney. All these examples are comprehensible only when the principle worked out by Daniel Bernoulli is understood.

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Dynamic lift on a spinning ball



Dynamic lift on air foil

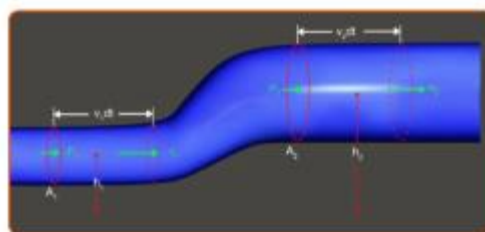
Bernoulli's principle explains that where the velocity of the fluid is high, the pressure is low, and where the velocity is low, the pressure is high.

Statement:

When a non- viscous, incompressible fluid flows steadily the sum of the pressure energy, kinetic energy and potential energy per unit volume of the fluid remains constant at all points in the path of the flow.

Now we derive the equation for Bernoulli principle.

Let us consider a non- viscous and incompressible fluid flowing steadily in a tube. The area of cross section of the end at point 1 is A_1 and it is at a height h_1 from the earth's surface. The area of cross section of the tube at point 2 is A_2 and is situated at a height h_2 ($h_2 > h_1$).



Bernoulli theorem

According to the equation of continuity $A_1v_1 = A_2v_2$. In a small time dt , the mass m of the fluid passing the point 1 is $\rho A_1v_1 dt$. If we assume the pressure at point 1 be P_1 , the work done by the fluid that enters the tube at that point is

$P_1A_1v_1dt$ and this work is positive because the force and displacement are in the same direction. If pressure at 2 is P_2 , then the work done on the fluid of mass ' m ' = (ρA_2v_2dt) at that point is $P_2A_2v_2dt$ and it is negative as force and displacement are in opposite directions.

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Therefore the work done on the fluid (due to pressure difference)

$$W_p = P_1 A_1 v_1 dt - P_2 A_2 v_2 dt. \dots\dots\dots(1)$$

The work done by the gravitational force on the fluid as the fluid flows from h_1 to h_2 is

$$W_g = m \times (h_1 - h_2) \times g. \dots\dots\dots(2)$$

Then the total work done is

$$W_p + W_g = P_1 A_1 v_1 dt - P_2 A_2 v_2 dt + m h_1 g - m h_2 g. \dots\dots\dots (3)$$

According to work energy theorem, work done is equal to change in KE.

$$\therefore P_1 A_1 v_1 dt - P_2 A_2 v_2 dt + m \times (h_1 - h_2) \times g = \frac{1}{2} m \times (v_2^2 - v_1^2). \dots (4)$$

Dividing both sides of the Eq. (4) by m

$= \rho A_1 v_1 dt = \rho A_2 v_2 dt$, the mass of the liquid that flows into or out of the tube in time dt ,

$$\text{We get, } \frac{P_1}{\rho} + h_1 g + \frac{1}{2} v_1^2 = \frac{P_2}{\rho} + h_2 g + \frac{1}{2} v_2^2 \dots\dots\dots (5)$$

$$\text{Or } \frac{P}{\rho} + h g + \frac{1}{2} v^2 = \text{constant.}$$

$$\text{Or } P + \rho h g + \frac{1}{2} \rho v^2 = \text{constant} \dots\dots\dots (6)$$

Equation (6) represents the Bernoulli principle and is called Bernoulli's equation.

for a horizontal streamline flow equation (6) reduces to

$$P + \frac{1}{2} \rho v^2 = \text{constant.} \dots\dots\dots (7)$$

Remembering that pressure is equal to pressure energy per unit volume, and ρ is the mass per unit volume, equation (7) says that in a horizontal streamline flow sum of pressure energy per unit volume and Kinetic Energy per unit volume is always constant.

Hence pressure is smaller at a point where velocity is greater and pressure is greater at point where velocity is small.

MECHANICAL PROPERTIES OF FLUIDS

Torricelli's law: The speed of efflux from an orifice of a tank is equal to that of the velocity acquired by a freely falling body from a height equal to that of the liquid above the orifice.

The word efflux means fluid outflow. Torricelli discovered that the speed of efflux from an open tank is given by a formula identical to that of a freely falling body.

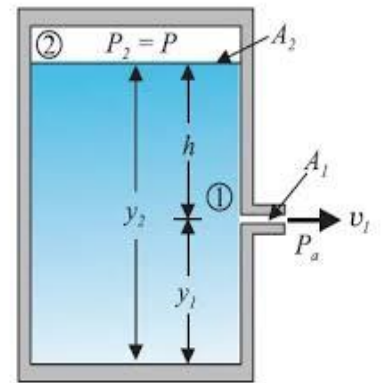
Consider a tank containing a liquid of density ' ρ ' with a small hole in its side at a height y_1 from the bottom.

From the equation of continuity, we have

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{A_1}{A_2} v_1$$

If the cross-sectional area of the tank A_2 is much larger than that of the hole ($A_2 \gg A_1$), then we may take the fluid to be approximately at rest at the top, i.e., $v_2 = 0$.



Now, applying Bernoulli's equation, at points (1) and (2) and noting that at the hole,

$$P_1 = P_a \text{ (atmospheric pressure)}$$

$$P_a + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

$$\Rightarrow \frac{1}{2} \rho v_1^2 = P - P_a + \rho g (y_2 - y_1)$$

Taking $y_2 - y_1 = h$, we have

$$v_1 = \sqrt{2g \square + \frac{2(P - P_a)}{\rho}}$$

Such a situation occurs in rocket propulsion.

On the other hand if the tank is open to the atmosphere, then $P = P_a$ and

$$v_1 = \sqrt{2g \square}$$

This is the speed of a freely falling body, at any point of height ' h ' during its fall.

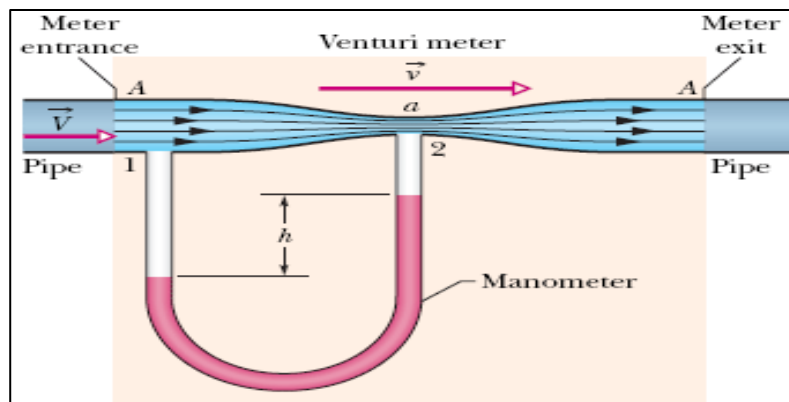
MECHANICAL PROPERTIES OF FLUIDS

This equation is known as Torricelli's equation.

Venturi-meter:

The venturi-meter is a device based on Bernoulli's theorem for measuring the rate of flow of an incompressible fluid through pipes.

It consists of a tube with a broad diameter and a small constriction at the middle.



A manometer in the form of a U-tube is also attached to it, with one arm at the broad neck point of the tube and the other at constriction the manometer contains a liquid of density ρ_m .

The speed v_1 of the liquid flowing through the tube at the broad neck area A is to be measured from equation of continuity.

Speed at the constriction becomes $v_2 = \frac{A}{a} v_1$

Then using Bernoulli's equation, we get

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

So that,

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

This pressure difference causes the fluid in the U-tube connected at the narrow neck to rise in comparison to the other arm.

The difference in height 'h' measures the pressure difference.

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$$P_1 - P_2 = \rho_m g \Delta h = \frac{1}{2} \rho v_1^2 \left[\left(\frac{A}{a} \right)^2 - 1 \right]$$

So that, the speed of fluid at wide neck is

$$v_1 = \sqrt{\left(\frac{2\rho_m g \Delta h}{\rho} \right) \left[\left(\frac{A}{a} \right)^2 - 1 \right]^{-1/2}}$$

Blood flow and heart attack:

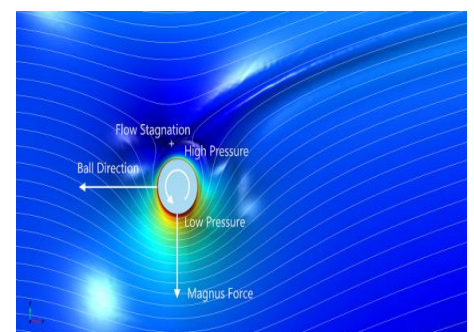
- Bernoulli's principle helps in explaining blood flow in artery.
- The artery may get constricted due to the accumulation of plaque on its inner walls.
- In order to drive the blood through this constriction a stress is placed on the activity of the heart.
- The speed of the flow of the blood in this region is raised which lowers the pressure inside and artery may collapse due to the external pressure.
- The heart exerts further pressure to open this artery and forces the blood.
- As the blood rushes through the opening, the internal pressure once again drops due to same reasons leading to a repeat collapse.
- This may result in heart attack.



Applications of Bernoulli's principle:

Dynamic lift on spinning ball:

- When a spinning ball is thrown it deviates from its usual path during flight. This effect is called Magnus effect. (It plays an important role in tennis, golf, cricket etc.)

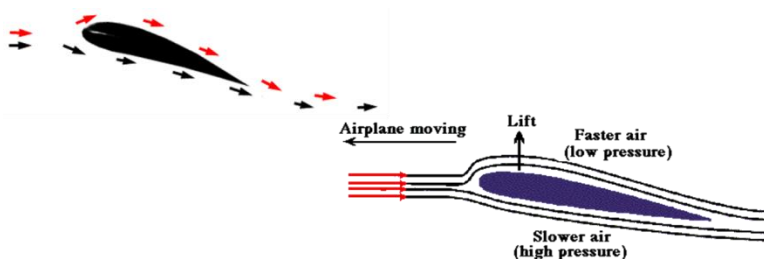
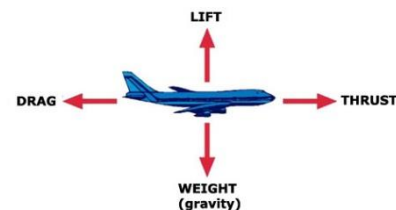


MECHANICAL PROPERTIES OF FLUIDS

- If the ball is moving in air without spinning, the stream lines around the ball are symmetric. Here $V_A = V_B$ and $P_A = P_B$
- If a ball is spinning, it drags some air with it. The distribution of streamlines around it changes as shown in the figure.
- For non-spinning ball $V_A = V_B \Rightarrow P_A = P_B$
- For spinning ball $V_A > V_B \Rightarrow P_A < P_B$
- Due to this pressure difference, the ball experiences an upward thrust called dynamic lift.
- If a ball is thrown with back spin, the pitch will curve less sharply prolonging the flight. If it is thrown with top spin the pitch will curve more sharply shortening the flight.
- If the ball is spinning about a vertical axis the curving will be sideways producing out swing or in swing.

Dynamic lift on aircraft wing or aerofoil:

- The front end of aerofoil or wing of an aeroplane is convex or round shaped and its thickness gradually tapers off from front edge to tailing edge.
- Due to this shape, the stream lines of air above the wing are closer than those below the wing.
- Then pressure above the wing will be less than the pressure below it. This pressure difference produces aerodynamic lift.



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- If surface area of wing of aeroplane is A , P_1 and P_2 are pressures on the top surface and on the bottom surface, then lift force on the wing is $F=(P_2-P_1)A$.
- If mg is weight of the wing, net force on the wing is $F-mg$.

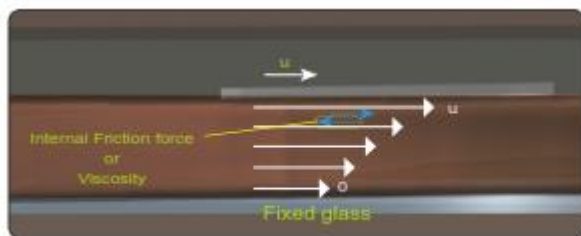
If V_1 and V_2 are the velocities of air above and below the wing,

$(P_2-P_1)=\frac{1}{2}\rho(V_1^2 - V_2^2)$ here ' ρ ' is density of air. If there is an obstacle in a tube of flow, the stream lines will split and become normal as shown. The points S_1 and S_2 are known as stagnation points. The velocity of liquid flow at stagnation point is zero.

VISCOSITY:

Viscosity is the property of fluid in motion. Most of the fluids are not ideal ones. They offer some resistance to the fluid motion. When the fluid moves in the form of layers having different velocity, tends to destroy the relative motion of the different layers of the fluid. This resistance is an internal friction of fluid, similar to the friction when a solid moves on the surface of another solid. This internal friction of moving fluid is called **viscosity of fluid**.

- Thus, viscosity is the property of the fluid (liquid or gas) by virtue of which an internal frictional force comes into play when the fluid is in motion in the form of layers having relative motion. It opposes the relative motion of the different layers. Viscosity is also called us **fluid friction**.
- To understand viscosity of fluid, take a fluid (i.e., liquid oil,) enclosed between two parallel glass plates as shown in figure.



- **Velocity distribution for viscous flow in a pipe**
- Keep the bottom plate fixed and move the top plate by applying some force with a constant velocity u , relative to the fixed plate. Replace the oil with honey, we will note that a greater force is required to move the top plate with the same velocity. It shows

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that the honey has more internal frictional force than oil, i.e., honey is more viscous than oil.

- In the above experiment, the layer of liquid in contact with the surface has the same velocity as that of the surface. It means, the layer of the liquid in contact with top glass plate moves with the velocity 'u' and the layer of liquid in contact with the fixed glass plate is stationary. The velocity of the different layers of liquid increases uniformly from bottom (zero velocity) to the top layer (velocity 'u').
- Every fast moving layer tends to pull forward its lower slow moving layer and every slow moving layer tends to pull backwards the upper fast moving layer. This produces a viscous force (called
- internal frictional force or viscosity) between the two layers of the liquid, which tends to destroy the relative motion of the two layers of liquid.
- Therefore, to maintain the relative velocity between the two layers of liquid flow, a force F (equal to the viscous force acting between these two layers) is to be applied tangentially on the liquid layer. Let initially BCDE be the portion of the liquid in between two parallel glass plates and l be the thickness of liquid. Let A be the area of the liquid in contact with glass plate.
- Let F be the tangential force applied to move the upper plate with a constant velocity u. After a short interval of time Δt , let the liquid take the shape FGDE as shown in Figure, where $BF = \Delta x = v \Delta t$.
- During this interval of time Δt , the liquid has undergone a shear strain of $\frac{\Delta x}{l}$. As the time passes, the shear strain in a flowing liquid increases continuously with time.
- Experimentally, it is found that in a flowing liquid the stress is proportional to the rate of change of strain or strain rate. It is different from solid, where stress is directly proportional to strain'.
- For the flowing liquid, the rate of change of strain or
- Strain rate = $\frac{\text{change in shear strain}}{\text{time interval}}$
- = $\frac{\frac{\Delta x}{l}}{\Delta t} = \frac{v}{l}$
- Shearing stress = $\frac{F}{A}$

MECHANICAL PROPERTIES OF FLUIDS

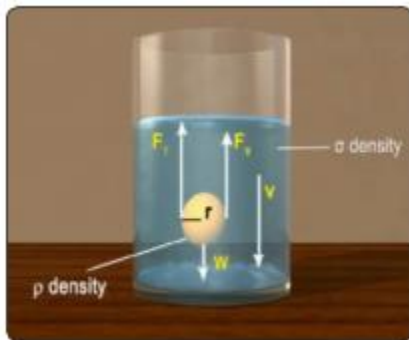
- The coefficient of viscosity ' η ' (for a fluid is defined as the ratio of shearing stress to the strain rate, i.e.,
- $\eta = \frac{\text{shearing stress}}{\text{strain rate}} = \frac{\frac{F}{A}}{\frac{v}{l}} = \frac{Fl}{vA} \rightarrow (1)$
- If ' dv ' is the difference in velocity of two layers of moving liquid distance ' dx ' apart, then
- $\frac{v}{l} = \frac{dv}{dx}$ = velocity gradient between two layers of liquid.
- Now $\eta = \frac{F}{A \frac{v}{l}} = \frac{F}{A \frac{dv}{dx}} \rightarrow (2)$
- If $A = 1$, $\frac{dv}{dx} = 1$ then from Equation (2)
- $\eta = \frac{F}{A(1 \times 1)} = F$
- This, coefficient of viscosity of a liquids equal to the tangential force required to maintain a unit velocity gradient between two parallel layers of liquid each of area unity.
- Dimensional formula of ' η '
- $\eta = \frac{Fl}{vA} = \frac{[MLT^{-2}][L]}{[LT^{-1}][L^2]} = ML^{-1}T^{-1}$
- Units of coefficient of viscosity
- From (2), ' η ' = $\frac{F}{A \frac{dv}{dx}}$
- The SI unit of coefficient of viscosity is Poiseuille (Pl) or Pa-S or
- N-s/m²
- Thus, coefficient of viscosity of a liquid is said to be 1 Pl or 1 Pa-s if 1 Newton tangential force is required to maintain the velocity gradient of 1 ms⁻¹/m, between two parallel layers of liquid each of area 1sq m.
- The CGS unit of coefficient of viscosity is dyne-s/cm²
- 1 poise = 10⁻¹N – s/m²
- **Distinction between viscosity and solid friction:**
- Viscosity is similar to the solid friction (friction among the solid surfaces in contact) in the respect that
 - (i) Both come into play whenever there is a relative motion,
 - (ii) Both oppose relative motion and

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- (iii) Both are due to molecular forces.
- (iv) Both act tangentially in a direction opposite to that of motion. Viscosity differs from the solid friction in the respect that the viscous force acting between two layers of the liquid depends upon
 - (i) The area of the layers,
 - (ii) The relative velocity of two layers and
 - (iii) Distance between two layers.
- (iv) The viscosity does not depend upon the normal reaction between two layers of the liquid. The friction between two solid surfaces is independent of the area of surfaces in contact and the relative velocity between them. The solid friction is directly proportional to the normal reaction between surfaces in contact.
- **Stoke's Law:**
- When a small spherical body moves through a viscous medium at rest, the layers of the medium touching the body are dragged along with it. But the layers of the medium away from the body are at rest. This causes a relative motion between different layers of medium.
- As a result of this, a backward dragging force (i.e., viscous drag) comes into play, which opposes the motion of the body. This backward dragging force increases with the increase in velocity of the moving body.
- Stoke found that the backward dragging force F acting on a small spherical body of radius r , moving through a medium of coefficient of viscosity η , with velocity v is
- Given by $F = 6\pi\eta rv \rightarrow (7)$
- This is called Stoke's law of viscosity.
- ***Importance of Stokes' law:***
- 1. This law is used in the determination of charge of an electron with the help of Millikan's experiment
- 2. This law accounts the formation of clouds.
- 3. This law accounts, why the speed of rain drops is less than that of a body freely fall with a constant velocity from the height of clouds
- 4. This law helps a man coming down with the help of a parachute.
- **TERMINAL VELOCITY:**

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- It is maximum constant velocity acquired by the body while falling freely in a viscous medium. When a small spherical body falls freely through a viscous medium, three forces act on it. Weight of the body acting vertically downwards, Upwards thrust due to buoyancy equals to weight of liquid displaced viscous drag acting in the direction opposite to the motion of the body. According to stroke's law $F \propto V$ i.e., the opposing viscous drag goes on increasing with the increasing velocity of the body .
- As the body falls through a medium, its velocity goes on increasing due to gravity, therefore the opposing viscous drag which acts upwards also goes increasing. A stage reaches when the true weight of the body is just equal to the sum of the upward thrust due to the buoyancy and the upward viscous drag. At this stage there is no net force to accelerate the body. Hence it starts falling with a constant velocity, which is called terminal velocity.
- Let ρ be the density of the material of the spherical body of radius r and σ be the density of the medium.

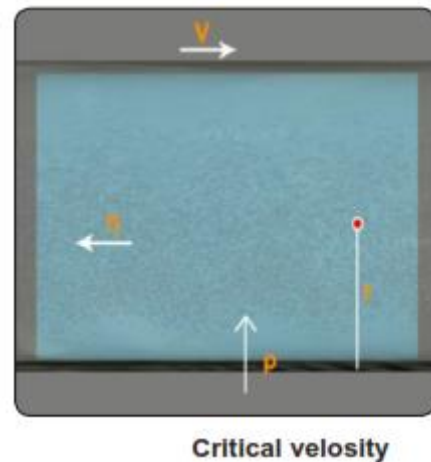


Terminal law

- \therefore True weight of the body,
- $W = \text{volume} \times \text{density} \times g = \frac{4}{3} \pi r^3 \rho g$
- Upward thrust due to buoyancy,
- $F_T = \text{weight of the medium displaced}$
- $F_T = \text{volume of the medium displaced} \times \text{density} \times g = \frac{4}{3} \pi r^3 \sigma g$
- If V is the terminal velocity of the body, then according to Stokes' law, upward viscous drag, $F_V = 6\eta\pi rV$ when body attains terminal velocity, then $F_T + F_V = W$
- $\frac{4}{3} \pi r^3 \sigma g + 6\eta\pi rV = \frac{4}{3} \pi r^3 \rho g$
- Or $6\eta\pi rV = \frac{4}{3} \pi r^3 (\rho - \sigma)g$

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- Or $V = \frac{2r^2(\rho - \sigma)g}{9\eta}$
- **Practical uses of the knowledge of the viscosity:**
- (i) The knowledge of viscosity and its variation with temperature helps us to select a suitable lubricant for a given machine.
- (ii) The knowledge of viscosity of some organic liquids such as proteins, cellulose etc helps us in determining their shape and molecular weight.
- (iii) At railway terminals, the liquids of high viscosity are used as buffers.
- (iv) Motion of some instruments is damped by using the viscosity of air or liquid.
- (v) The knowledge of viscosity helps in determining charge on an electron.
- (vi) The phenomenon of viscosity plays an important role in the circulation of blood through arteries and veins of human body.



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Surface tension :

Introduction:

Some insects, like water spiders walk on water surface as if they are walking on a stretched elastic membrane. Their feet produce 'dimples' on the surface film without



rupturing the film.

The bristles of a paint brush spread out when the brush is immersed in water. However, when the brush is taken

These are due to

Surface tension is a state which will be

Surface tension is a electromagnetic

discuss about molecular theory related to this concept.



out, the bristles cling together.

phenomenon called surface tension.

one of the properties of matter in liquid discussed in this chapter

molecular phenomenon. It is due to the force between the molecules. Let us

Molecular theory: The forces between the molecules are of two types:

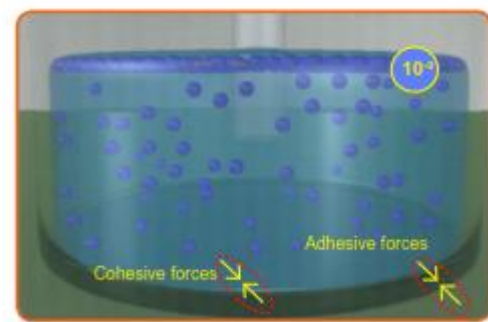
- i) Cohesive forces and
- ii) Adhesive forces.

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The force of attraction between the molecules of the same substance is called cohesive force.

The force of attraction between the molecules of different substances is called adhesive force.

The cohesive force becomes negligible beyond a distance of 10^{-9} m which is equal to 10 to 15 molecular diameters. The maximum distance upto which the force of attraction between two molecules can act is called molecular range. It is of the order 10^{-9} meters and it is different for different substances. A sphere drawn with a molecule as center and molecular range as radius is called sphere of influence. A molecule attracts all the molecules inside the sphere of influence and the molecule at the center will be attracted by all those molecules inside the sphere of influence.



Molecular theory

Molecular Theory of Surface Tension:

We can explain the surface tension by using molecular theory.

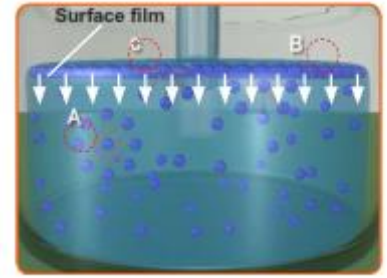
Consider three molecules A, B and C of a liquid as shown in the figure. The molecule A has its sphere of influence completely inside the liquid. Another molecule B is within the surface film which is the layer of the liquid present between the surface of the liquid and a plane drawn at a distance of a molecular range from the surface. Its sphere of influence is partly outside the liquid. The third molecule C is on the surface of the liquid so that half of its sphere of influence is outside the liquid.

Let us discuss about the molecule A: As the surface of influence of the molecule A is completely inside the liquid, the cohesive forces on all sides of the molecule balance each other. Hence, there is no resultant force on the molecule A.

Let us discuss about the molecule B: The lower half of the sphere of influence of the molecule B is completely inside the liquid. Part of the upper half of the sphere of influence of the molecule B is outside the liquid. So, the number of molecules in the lower half of the sphere of influence is greater than the number of molecules in the upper half of the sphere of influence and there is a resultant downward force on the molecule B.

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Let us discuss about the molecule C: In the lower half of the sphere of influence of the molecule C is completely inside the liquid whereas the upper half of the sphere of influence is completely outside the liquid. Hence, there is maximum downward force on the molecule C.

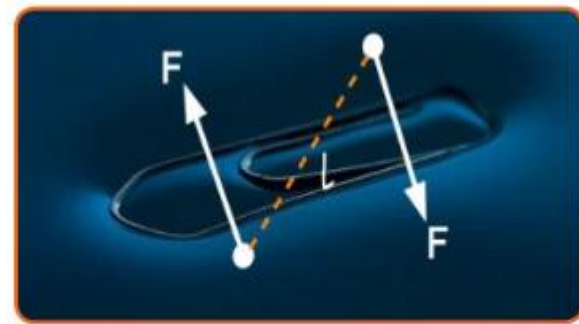


Molecular theory of surface tension

The molecules in the surface film and in the surface of the liquid experience downward force with maximum downward force on the molecules of the surface. Due to this downward force the surface of the liquid behaves like a stretched elastic membrane which is the cause of surface tension.

Surface Tension Definition:

According to molecular theory, it is observed that the surface of a liquid behaves like a stretched elastic membrane or a stretched rubber sheet. A rubber sheet is in the state of tension when it is stretched from all sides. Any part of the rubber sheet pulls the adjacent part proportional to the length of the line. Similarly the part of the surface on the right side pulls the part of the surface on the left side with an equal and opposite force. The force, F is perpendicular to the line and tangential to the surface as shown in figure.



surface tension force

Surface tension is defined as the tangential force per unit length, acting at right angles on either side of the line, imagined to be drawn on the free liquid surface in equilibrium.

If ' F ' is the force acting perpendicular to the imaginary line of length ' l ', then by the definition of surface tension.

$$\text{Surface tension} = \frac{\text{force}}{\text{length}}$$

$$T = \frac{F}{l}$$

Its SI unit is 'Newton per meter'. Its CGS unit is 'dyne per centimeter'.

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Its dimensional formula: $\left[\frac{MLT^{-2}}{L}\right] = [ML^0T^{-2}]$

Examples for the phenomenon of Surface Tension:

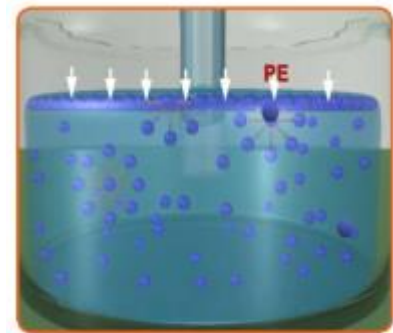
i) A small drop of a liquid assumes spherical shape, since for a given volume a sphere has minimum surface area.

ii) A blotting paper, with a greased needle on it, is placed on the surface of water. When the blotting paper absorbs water it sinks leaving the greasy needle on the surface. The greasy needle floats on the surface of the water. It will be observed that the surface of the water below the needle is slightly depressed like that of the depression of a rubber sheet on which a ball is kept. The weight of the needle is balanced by the inclined upward tension forces as shown in diagram.

Surface Energy:

Let us discuss about the surface energy.

We have seen that the molecules whose sphere of influence is completely inside the liquid have no resultant force due to the molecular forces. The molecules in the surface film and the molecules on the surface experience resultant downward force. This force tries to pull the molecules in to the liquid and new molecules from inside go to the surface and fill the space. These new molecules do work against the downward force which is stored as potential energy of the molecules. Thus the molecules in the surface film and on the surface have additional potential energy due to the molecular forces.



surface energy

Definition of Surface Energy (E):

The additional potential energy due to the molecular forces per unit surface area is called surface energy (or) The excess potential energy per unit area of the surface film is called surface energy.

$$\text{Surface energy} = \frac{\text{Potential energy due to molecular forces}}{\text{Surface area}}$$

Its SI unit is 'Joule per meter square'. Its CGS unit is 'erg per centimeter square'.

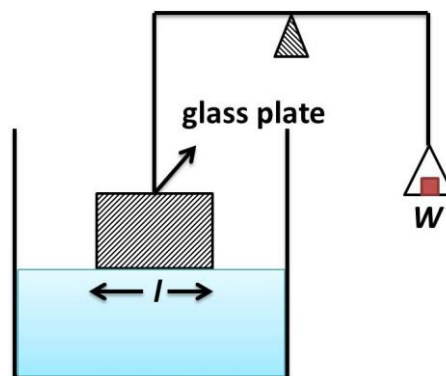
MECHANICAL PROPERTIES OF FLUIDS

Its dimensional formula = $\left[\frac{ML^2T^{-2}}{L^2}\right] = [ML^0T^{-2}]$

A system in equilibrium has a tendency to acquire lowest possible potential energy. For minimum potential energy in the surface film, minimum number of molecules must be present in it. This is possible by decreasing the surface area of the surface film. Hence, the free surface of a liquid always tends to have minimum possible area which is a consequence of surface tension.

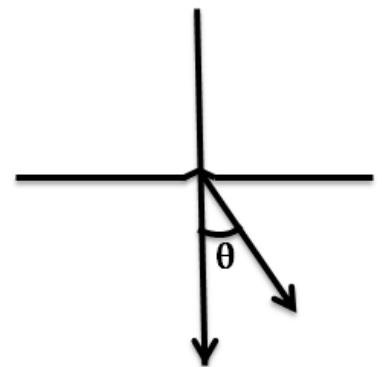
EXPERIMENT TO DETERMINE SURFACE TENSION:

Surface tension of a liquid can be directly measured experimentally. A flat vertical glass plate, below which a vessel of some liquid is kept, forms one arm of the balance. The plate is balanced by weights on the other side, with its horizontal edge just over water. The vessel is raised slightly till the liquid just touches the glass plate and pulls it down a little because of surface tension. Weights are added till the plate just clears water. Suppose the additional weight required is W . Surface tension of liquid-air interface is $T_{la} = \frac{W}{2l} = \frac{mg}{2l}$ where l is length of the plate edge and m is extra mass.



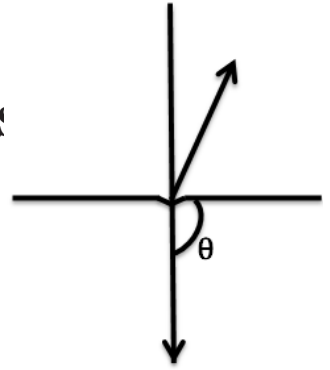
ANGLE OF CONTACT

It is generally observed that when a liquid surface comes in contact with a solid surface, the shape of the liquid surface near the place of contact is curved. If a glass plate kept vertically in water the surface of water is slightly raised all around the glass plate. i.e., the surface becomes concave if the water is pulled up as shown in figure.



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Similarly, a glass plate kept vertically in mercury, the surface of mercury is slightly depressed all around the glass plate i.e., the surface becomes convex if the mercury is pulled down as shown in figure.



Definition of angle of contact: The angle between the tangent to the liquid surface at the point of contact and the solid surface. Inside the liquid is called the angle of contact for a pair of solid and liquid.

If the liquid rises up along the solid surface the angle of contact is acute i.e., less than 90° e.g., glass plate in water, glass plate in kerosene, glass plate in alcohol.

If the liquid is depressed along the solid surface, the angle of contact is obtuse i.e., greater than 90° . E.g., glass plate in mercury, water with paraffin, water with water proof material.

If the angle of contact is acute, the liquid wets the solid. If the angle of contact is obtuse, the liquid does not wet the solid.

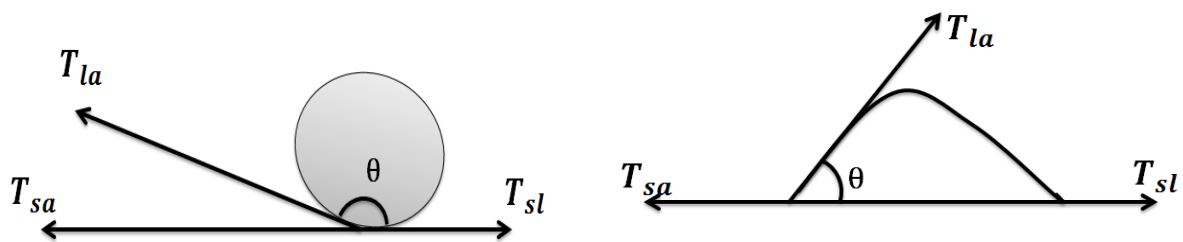
The angle of contact depends

- a) The nature of liquid-solid pair under consideration
- b) The medium above the free surface of the liquid
- c) The cleanliness and freshness of the two surfaces in contact.
- d) The temperature.
- e)
 - i) The general angle of contact decreases with increase in temperature.
 - ii) Angle of contact decrease with partially soluble impurities, like detergents and wetting agents.
 - iii) Angle of contact increases with soluble impurities like NaCl.
 - iv) Angle of contact increases with addition of water proofing agent.
 - v) Angle of contact does not depend upon angle of inclination.

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When a liquid and a solid are brought in contact with each other, the liquid surface generally becomes curved near the region of contact. The curvature is due to two forces, forces of cohesion between liquid molecules and adhesion between solid and liquid molecules. The nature and magnitude of the curvature is determined by the relative magnitude of the two forces.

We consider the three interfacial tensions at all three interfaces, liquid-air, solid-air and solid-liquid denoted by T_{la} , T_{sa} & T_{sl} respectively.



From fig. $T_{la} \cos \theta + T_{sl} = T_{sa}$

The angle of contact is an obtuse angle ($\theta > 90^\circ$) is $T_{sl} > T_{la}$ as in the case of water leaf interface while it is an acute ($\theta < 90^\circ$) angle $T_{sl} < T_{la}$ as in the case of water-plastic interface. When θ is an obtuse angle then molecules of liquids are attracted strongly to themselves and weakly to those of solid, it costs a lot of energy to create a liquid-solid surface, and liquid then does not wet the solid. This is what happens with water on a waxy or oily surface and with mercury on any surface. On the other hand, if the molecules of the liquid are strongly attracted to those of the solid, this will reduce T_{sl} and therefore, $\cos \theta$ may increase or θ may decrease. In this case θ is an acute angle. This is what happens for water on glass or on

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plastic and for kerosene oil on virtually anything (it just spreads). Soaps, detergents and dyeing substances are wetting agents. When they are added the angle of contact becomes small so that they may penetrate well and become effective. Water proofing agents on the other hand are added to create a large angle of contact between the water and fibers.

DETERGENT OR WETTING AGENT

Wetting agent is a material, mixed with liquid, to decrease the angle of contact with the given solid.

When washing soap is mixed with water, the angle of contact decreases. The cloths are soaked well in soap water than in ordinary water. Thus the cleaning of cloths is more effective with soap water. When a wetting agent is mixed with the colour to be dyed on fabric material, the angle of contact decreases and the dye would 'creep' deep into fabric material. Thus the colouring is uniform and lasts longer on the fabric.

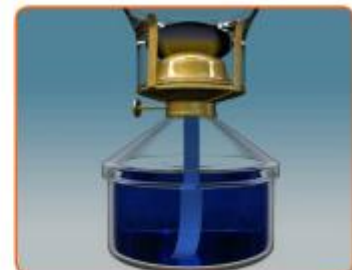
WATER PROOFING AGENT

Water proofing agent is a material applied on the surface of solid to increase the angle of contact (angle made obtuse) with water.

While preparing water proofing dress, the fabric is coated with wax. As such, the angle of contact between water and wax-coated fabric becomes obtuse ($>90^\circ$). When this dress is used in rain, water does not wet the dress but rolls down as drops.

Capillarity:

When a capillary tube (capillary tube is a tube of very fine bore) open at both ends is dipped



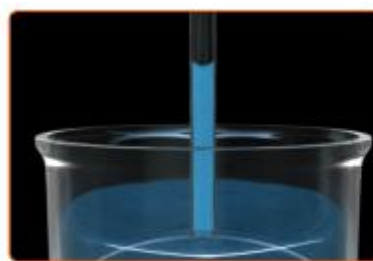
Capillarity

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into a liquid vertically, the liquid inside the capillary tube either rises or falls due to surface tension. This phenomenon is called capillary action. This is also called capillarity.

If the angle of contact is less than 90° , the liquid rises. If the angle of contact is greater than 90° , the liquid falls.

When a glass capillary tube is dipped vertically in a liquid of angle of contact less than 90° , the liquid rises in the tube until the force due to surface tension is balanced by the weight of the liquid lifted up. When the angle of contact is greater than 90° , the liquid gets depressed due to surface tension forces acting downwards.



Capillary rise



Capillary down

Examples for capillarity in daily life:

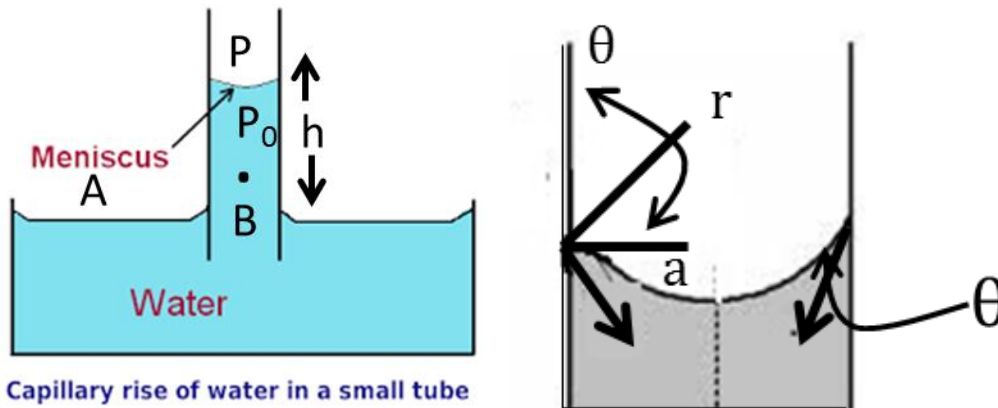
1. **Working of kerosene lamp:** The capillaries in a wick allow the kerosene to rise up in a kerosene lamp or a kerosene stove so that the kerosene can be burnt continuously to produce a flame.
2. **Working of Candle:** The capillaries in a wick allow the paraffin wax to rise up when the paraffin wax melts while the candle flame is produced.
3. **Cotton clothes:** When we wear cotton clothes during summer, the sweat from the skin is removed by the fine pores in the cloths due to capillary action.
4. **Blotting paper:** Blotting paper is used to absorb ink. The ink rises through the capillaries in the blotting paper due to capillarity.
5. **Roots in plant:** Sap rises up to the various parts of the plant through the capillaries present in the plant by capillary action.
6. **Sponge:** A sponge has very small pores which act as small capillaries, that is why a sponge can absorb a comparatively large amount of liquid.

Determination of surface tension by capillary rise method:

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Consider a vertical capillary tube of circular cross section (radius a) inserted into an open vessel of water (Fig.). The contact angle between water and glass is acute. Thus the surface of water in the capillary is concave. This means that there is a pressure difference between the two sides of the top surface.

This is given by $(P_i - P_0) = (2T/r) = 2T/(a \sec\theta) = \frac{2T \cos\theta}{a}$



Thus the pressure of the water inside the tube, just at the meniscus (air - water interface) is less than the atmospheric pressure P_0 . Consider the two point A and B in Fig(a). They must be at the same pressure,

$P_0 + h\rho g = P_i = P_A$ where ρ is the density of water and h is called the capillary rise. Then $h\rho g = (P_i - P_0) = (2T \cos\theta)/a$ so it is clear that the capillary rise is due to surface tension. It is larger, for a smaller a .

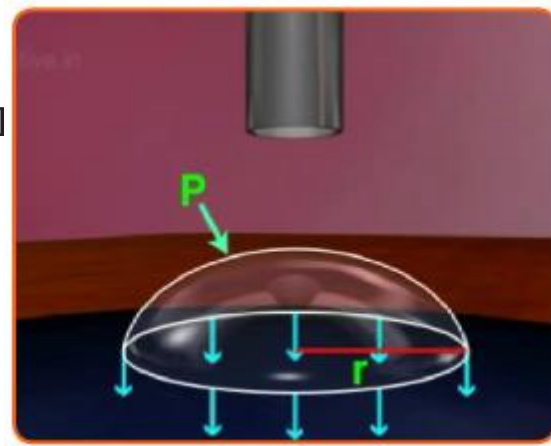
EXCESS OF PRESSURE:

Excess of pressure in a liquid drop:

In the case of a spherical liquid drop, the net forces on the molecules present near the surface of a liquid drop are subjected to resultant inward pull due to surface tension. Hence, the pressure inside the drop will be greater than pressure outside.

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Assume the excess pressure to be 'P' and the radius of the drop 'r'. 'T' represents the surface tension of the liquid. Let us consider equilibrium along any diametrical plane. The surface tension acts along the rim of the length $2\pi r$, as shown in the figure. Force due to excess pressure P acts on area πr^2 . Under



Liquid Drop

equilibrium, the upward force acting due to the excess pressure 'P' on the upper hemisphere = the downward force acting along the circumference due to surface tension of the liquid.

$$\text{i.e. } P(\pi r^2) = 2\pi r(T) \quad \text{or } P = \frac{2T}{r}$$

Excess of pressure in a soap bubble:

It has two free surfaces, one inside and the other outside. Taking the thickness of the film to be very small as compared to the radius of the bubble, the surface tension force along the two edges of the diametrical plane will be under equilibrium condition.

$$P(\pi r^2) = 4\pi r(T)$$

$$\text{That implies, } P = \frac{4T}{r}$$

It is evident from the above equation that excess pressure is inversely proportional to the radius of the drop. The excess pressure inside a small drop will be greater than that of a big drop.

Shape of a liquid drop:

The shape of a liquid drop depends on two forces.

- i) Gravitational force, and
- ii) Force of surface tension.

The condition for the shape of the drop in equilibrium is that the total potential energy, i.e., Gravitational potential energy + potential energy due to surface tension.

Surface tension forces are considered, the potential energy will be minimum for minimum surface area i.e., the drop must be of spherical shape. If only gravitational force is considered, the potential energy will be minimum when the centre of gravity of the drop is at the least height i.e., the drop must be flat.



Soap Bubble

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When we consider both the gravitational force and the force of surface tension, for a small drop the force due to surface tension predominates and hence it assumes spherical shape. For a large drop, the gravitational force predominates and becomes flat there by lowering the centre of gravity and potential energy to the least possible.

If a drop of oil is placed in a mixture of water and alcohol of same density as that of the oil, the gravitational force on the drop will be balanced by upward thrust on the drop of oil due to the mixture of water and alcohol since both the densities are equal. So only the force due to surface tension remains and the drop acquires a shape of perfect sphere.

If a liquid drop of lower density is placed on the liquid surface of higher density, the liquid drop spreads on the surface of the liquid of higher density. For example, when a drop of pure water is placed on the surface of mercury, it spreads over it and forms a uniform thin layer.

VARIATION OF SURFACE TENSION WITH TEMPERATURE

When temperature of a liquid increases, the kinetic energy of its molecules increases, and the cohesive forces become weak. As such, whenever the temperature of liquid increases, its surface tension in general, decreases. The variation of surface tension of liquid with temperature (for small range) can be expressed as, $S_1 = S_0(1 - \alpha t)$, where S_0 and S_1 are surface tensions of liquid at 0°C and $t^\circ\text{C}$ respectively. 'a' is a constant, called temperature coefficient of surface tension. The value of 'a' is different for different liquids.

At critical temperature there is no demarcation between of the liquid phase and gaseous phase of matter. Hence at critical temperature the interface between the liquid and its vapour disappears. It is found experimentally that the surface tension of any liquid becomes zero at a temperature slightly less than its critical temperature.

The surface tension of molten cadmium and molten copper increases with increase of temperature.

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EFFECT OF IMPURITIES ON SURFACE TENSION

1) *If the liquid surface is contaminated by impurities, its surface tension decreases immediately and sharply.*

Ex. When oil, grease etc are sprayed on water surface, the surface tension of water decrease sharply.

2) *If the added impurities are highly soluble in liquid, its surface tension increases*

Ex: When highly soluble salts like NaCl, ZnSO₄ etc are mixed with water, its surface tension increases.

3) *If the added impurities are weakly soluble in liquid, its surface tension decreases.*

Ex: When soap is mixed with water, the surface tension decreases and washing is more effective.